About the Origin of Cities

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Systems of cities and urban hierarchies

- Despite big variations in the natural landscape, urban systems exhibit strikingly regular hierarchical structures
  - Marshall (1989); Hsu (2012), Hsu et al. (2014)

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  What makes these structures to emerge?

- Question 2:

  How can these structures be rationalized?

- Answers: two competing theories

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Central place theory vs New economic geography

Central place theory (Christaller, 1933; Lösch, 1940):

- Makes strong predictions on spatial distribution of economic activities
- Problem: lacks solid micro-foundations ☹

New economic geography (Krugman, 1991; Fujita et al., 1999):

- Relies on a full-fledged general equilibrium setup
- Problem: two-region setting $\Longrightarrow$ no urban hierarchy ☹

Wanted: a reconciliation

What we do

- Provide a bare–bones spatial framework generating:
  - either equally-spaced and equally-sized central places
  - or a hierarchy of central places
- Derive spatial patterns from individual decisions based on:
  - preferences of agents
  - efficiency of communication technologies
- One type of agents $\Longrightarrow$ periodic distribution of cities
- Two types of agents $\Longrightarrow$ urban hierarchy

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Homogeneous world: one type of agents

Spatial structure

- Space is assumed to be:
  - one-dimensional
  - featureless
  - unbounded
- We represent the space as the real line $\mathbb{R}$ with locations $x, y \in \mathbb{R}$
- There is a continuum of agents distributed over $\mathbb{R}$
- All agents are identical in that they conduct the same type of activity
- The population density $n(x) \geq 0$ satisfies the following condition:

$$\overline{n} \equiv \lim_{b \to \infty} \left[\frac{1}{2b} \int_{-b}^{b} n(x) \mathrm{d}x \right] < \infty$$

Preferences

- Two opposing forces:
  - benefits from communication — centripetal force
  - cost of congestion — centrifugal force
- Linear utility function:

$$u(x) = E(x) - \alpha n(x)$$

- $E(x)$ = spatial externality (to be explained below)
- $n(x)$ = population density at the place x of agent's residence
- α = congestion cost per unit of population density ($\alpha > 0$)

Spatial externality

- Agents are embedded into a social interaction field
  - see, e.g., Jackson et al. (2017)

- Social interaction decays with distance:

$$\text{Interaction between } x \text{ and } y = \varphi(|x-y|), \quad \varphi'(\cdot) < 0$$

- Interaction flow from y to x:

$$\varphi(|x-y|)n(y)$$

- Spatial externality is my total interaction with all the others:

$$E(x) \equiv \int_{\mathbb{R}} \varphi(|x-y|)n(y)\mathrm{d}y$$

Distance decay function

- Assume that distance decay of social interaction is exponential:

$$\varphi(|x-y|) = \exp\{-\beta\,|x-y|\}$$

- This specification has been widely used in the literature:
  - Fujita and Ogawa (1982)
  - Lucas and Rossi-Hansberg (2002)
  - Desmet and Rossi-Hansberg (2013)

- The parameter $\beta > 0$ is a measure of spatial frictions
- When x and y are close to each other, we have:

$$\exp\{-\beta\,|x-y|\} \approx 1-\beta\,|x-y|$$

Spatial equilibrium

- Equilibrium $\Longleftrightarrow$ agents have the same utility level u^* at all locations:

$$u(x) = u^* \qquad \text{for all } x \in \mathbb{R}$$

- Intuition: equilibrium is an outcome when nobody wants to move
- Linear utility and exponential distance decay yield:

$$n(x) = \frac{1}{\alpha}\left[\int_{\mathbb{R}} \exp\{-\beta |x-y|\} n(y)dy - v^*\right]$$

- This equation is:
  - a linear integral equation
  - known as the Fredholm equation of 2nd kind

Spatial frictions

- Two sources of spatial frictions:
  - disutility α of congestion
  - inefficiency β of communication technology
- We merge the two into a sole parameter:

$$\phi \equiv \frac{2}{\alpha\beta}$$

- Intuitively, ϕ is an inverse measure of spatial frictions

Flat world vs equidistant equal-sized settlements

Proposition 1:

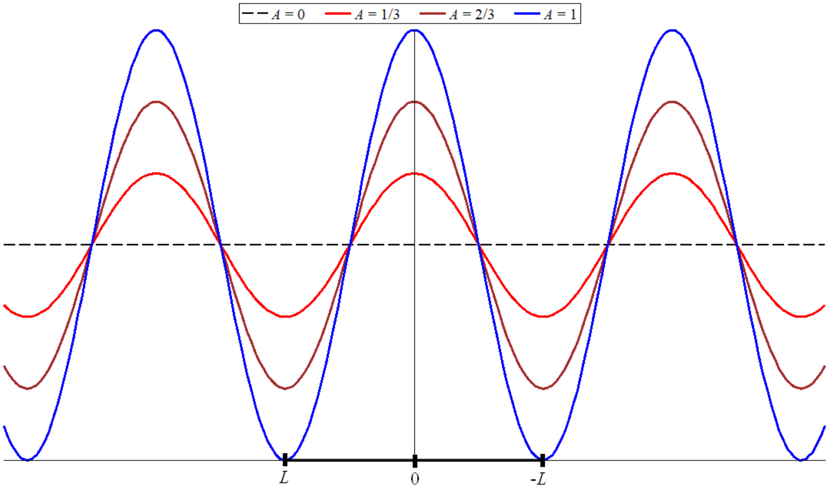
- $\phi \leq 1 \implies$ only uniform equilibrium

Proposition 2:

- $\phi > 1 \implies$ non-uniform equilibria:

$$n^*(x) = \overline{n} + A \sin\left(\beta\sqrt{\phi - 1}\, x\right), \qquad |A| \leq \overline{n}$$

Equidistant equal-sized settlements $\iff \phi > 1$



Stable equilibrium

Proposition 3. When $\phi > 1$, there exists a unique stable spatial equilibrium given by

$$n^*(x) = \overline{n}\left[1 + \sin\left(\beta\sqrt{\phi - 1}\,x\right)\right]$$

Does theory meet historical evidence?

- Early times: $\phi < 1$ was satisfied
- People remained dispersed because:
  - land could not support agglomeration
  - encounter with strangers was dangerous
- Gradually, the world became uneven:
  - food production $\Longrightarrow$ higher land returns $\Longrightarrow \alpha \downarrow$
  - specialization $\Longrightarrow$ need to interact closer $\Longrightarrow \beta \downarrow$
- Lower spatial frictions $\Longrightarrow \phi > 1 \Longrightarrow$ CENTRAL PLACES!

Welfare

Proposition 4.

- Equilibrium welfare level:

$$u^* = \overline{n}\left(\frac{2}{\beta} - \alpha\right)$$

- Urbanization-enhancing shock = welfare-enhancing shock!
  - living together is less costly $\implies \alpha \downarrow \implies u^* \uparrow$
  - transportation improvement $\implies \beta \downarrow \implies u^* \uparrow$

- Higher average population $\overline{n}$ per unit of land:

Welfare increases/decreases $\iff \phi \gtrless 1$

A model with two types of agents

Two types of agents

- Assume now that there are two populations of agents
- For example, these can be:
  - hunters and gatherers
  - farmers and artisans
  - firms and consumers
- Agents differ across — not within! — populations in:
  - preferences
  - communication technologies
- Each agent:
  - gains from communication with agents of both types
  - suffers from congestion with agents of both types

Two types of agents

- Density of population j:

$$n_j(x) \geq 0, \qquad j = 1, 2$$

- Spatial externality a j-type agent at $x \in \mathbb{R}$ receives from k-type agents:

$$E_{jk}(x) \equiv \int_{-\infty}^{\infty} \exp\{-\beta_{jk}|x-y|\} n_k(y) \mathrm{d}y$$

- Utility of a j-type agent:

$$u_j(x) = \gamma_{jj} E_{jj}(x) + \gamma_{jk} E_{jk}(x) - \alpha_{jj} n_j(x) - \alpha_{jk} n_k(x)$$

- Population-specific preferences are now captured by twelve parameters:

$$\alpha_{jk} > 0, \quad \beta_{jk} > 0, \quad \text{and} \quad \gamma_{jk} > 0, \qquad j, k = 1, 2$$

Quasi-symmetric world

Consider the following quasi-symmetric case:

- Each agent only gets congested with her own people:

$$\alpha_1 \equiv \alpha_{11} > 0, \qquad \alpha_2 \equiv \alpha_{22} > 0, \qquad \alpha_{12} = \alpha_{21} = 0$$

- Same intensity of the spatial externality:

$$\beta \equiv \beta_{11} = \beta_{12} = \beta_{21} = \beta_{22} > 0$$

- Same communication preferences within and across populations:

$$\gamma_{11} = \gamma_{22} = 1, \qquad \gamma \equiv \gamma_{12} = \gamma_{21} > 0$$

Equilibria in a quasi-symmetric world

Proposition 5. Assume a quasi-symmetric world in which $\gamma < 1$, i.e. each agent prefers interacting with her own people. Then:

- high spatial impedance $\beta \Longrightarrow$ only uniform equilibrium
- intermediate spatial impedance $\beta \Longrightarrow$ central places of the same size
- low spatial impedance $\beta \Longrightarrow$ urban hierarchy: non-uniform equilibria with central places of different sizes

Asymmetric world

Let $\mathbf{D}$ be a (4×4)-matrix independent of x defined as follows:

$$\mathbf{D} \equiv \mathbf{Q} - 2\mathbf{B}\mathbf{A}^{-1}\mathbf{\Gamma},$$

where

$$\mathbf{Q} \equiv \begin{pmatrix} \beta_{11}^2 & 0 & 0 & 0 \\ 0 & \beta_{12}^2 & 0 & 0 \\ 0 & 0 & \beta_{21}^2 & 0 \\ 0 & 0 & 0 & \beta_{22}^2 \end{pmatrix}, \quad \mathbf{B} \equiv \begin{pmatrix} \beta_{11} & 0 \\ 0 & \beta_{12} \\ \beta_{21} & 0 \\ 0 & \beta_{22} \end{pmatrix}$$

$$\mathbf{\Gamma} \equiv \begin{pmatrix} \gamma_{11} & \gamma_{12} & 0 & 0 \\ 0 & 0 & \gamma_{21} & \gamma_{22} \end{pmatrix}, \qquad \mathbf{A} \equiv \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$$

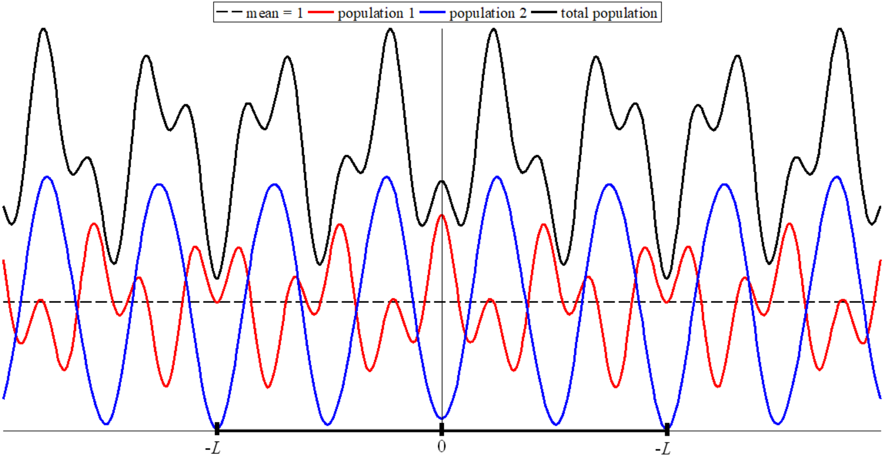
Equilibria in an asymmetric world

Proposition 6.

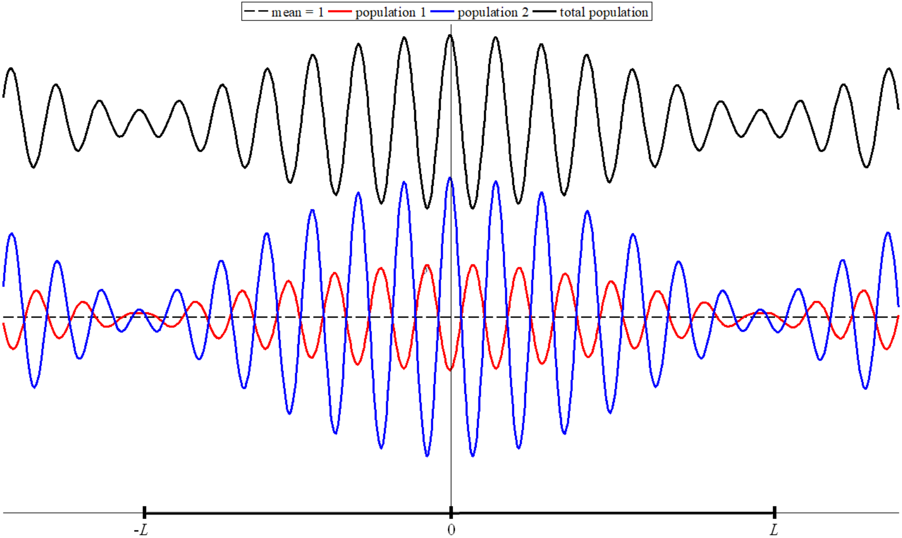
- D has no strictly negative real eigenvalues $\Longrightarrow$ only uniform equilibrium
- D has one strictly negative real eigenvalue $\Longrightarrow$ equidistant equally sized central places
- D has at least two strictly negative real eigenvalues $\Longrightarrow$ spatial equilibria involve different extrema

Urban hierarchy: illustrations

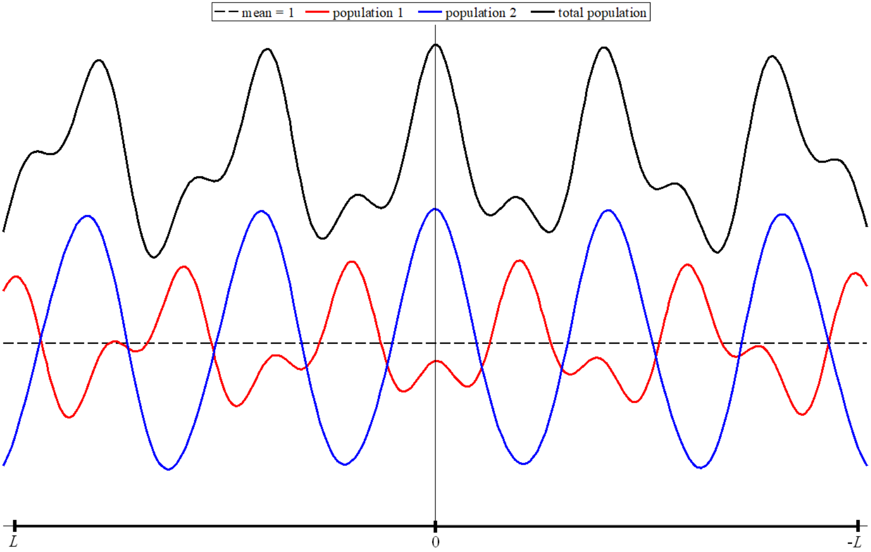
Population densities in a quasi-symmetric world



Christaller-like population densities



The "Big Five" in a heterogeneous world



Thank you for your attention!